Mechanism Design

- Games of Mechanism Design are an important class of games
- There is 1 special player called „principal”, who’s task is to set the rules of the „game”, and I other players called „agents”.
- Principal’s utility depends on some private information, which is held by the agents, but the agents are not willing to report that information unless given proper incentives

Formalization

- The principal has to determine an allocation $y = \{x, t\} \in Y$
- The vector x (in Euclidean space) is called „decision”, and the vector t are „transfers” to (+) or from (-) each agent. Both x and t are I-dimensional.
- Each agent i maximizes $u_i(y_i, \theta_i)$, where θ_i is her privately-known type
- First, the principal designs and announces a „mechanism”, which is a function $M \to Y$, where M is the set of messages sent by agents. Then agents send messages μ_i, such that $\mu = (\mu_1, ..., \mu_I) \in M$

Mechanism Design- Examples

Application	x	t	θ
Price discrimination	Amount purchased	Payment for purchase	Consumer's valuation of the good
Regulation	Firm's price	Firm's income	Firm's cost
Income tax	Personal income	Amount of tax	Ability to earn money
Public good	Amount of public good	Consumers' contributions to budget	Private benefits from public good
Auctions	Probability that consumer buys (wins)	Amount paid	Private valuations of the good

Revelation principle

- Take any mechanism that has a Bayesian equilibrium, in which the agents send messages μ^* the outcome function is y_m. There is a corresponding direct mechanism in which the agents decide to participate, submit their true types, and the allocations are the same as in the above equilibrium. In that direct mechanism, the principal calculates μ^* based on the submitted valuations and determines the outcome using y_m.
- In the direct mechanism, the principal says: tell me your true θ_i *'s,* I will determine the outcome based on the messages that these types would send in equilibrium.
- This means that we can limit our attention to direct mechanism, which simplifies the analysis

Application – Non-linear pricing

- There is one (potential) buyer with 2 types of $\theta = \{h, l\}$ with probabilities $(\lambda, (1-\lambda))$;
- Both have quasilinear demand functions
  $U^\theta = u(x, \theta) - t^\theta$
  $u(x, h) \geq u(x, l)$ for all x
- The monopolist will offer two allocations (x^l, t^l) and (x^h, t^h), and the buyer will send a signal: accept one of them or reject both.
- The monopolist has zero costs.

NLP – cont.

- The principal (seller) maximizes:
  max $N[\lambda t^l + (1-\lambda)t^h] = \max \lambda t^l + (1-\lambda)t^h$
- s.t. Individual rationality (participation) and Incentive compatibility constraints
- (IR^l) $t^l \leq u(x^l, l)$
- (IR^h) $t^h \leq u(x^h, h)$
- (IC^l) $u(x^h, l) - t^h \leq u(x^l, l) - t^l$
- (IC^h) $u(x^l, h) - t^l \leq u(x^h, h) - t^h$

NLP – cont.

- (IR^l) $t^l \leq u(x^l, l)$
- (IR^h) $t^h \leq u(x^h, h)$
- (IC^l) $t^l \leq u(x^l, l) - u(x^h, l) + t^h$
- (IC^h) $t^h \leq u(x^h, h) - u(x^l, h) + t^l$
- Also, assume $x^l \neq x^h$, and x^l, $x^h > 0$
- Notice that cetris paribus, we want to raise t^l until either (IR^l) or (IC^l) becomes binding.
- Notice that cetris paribus, we want to raise t^h until either (IR^h) or (IC^h) becomes binding.
- Can (IR^h) be binding? Then (IC^h) becomes $t^l \geq u(x^l, h)$. Combine that with (IR^l) we conclude that $u(x^l, l) \geq t^l \geq u(x^l, h)$ or $u(x^l, l) \geq u(x^l, h)$. But this contradicts our assumption.

NLP – cont.

- So (IR^h) cannot be binding → (IC^h) must be binding (type H is indifferent between buying x^h and x^l.)
- Suppose (IC^l) is binding.Then from the binding (IC^h) we substitute into (IC^l) and get $u(x^h, h) - u(x^l, h) = u(x^h, l) - u(x^l, l)$ → this is ruled out by the **non-crossing assumption**
- Hence (IR^l) must be binding →(**no surplus at the bottom)**
- The maximization problem becomes:

$$\max \lambda\, u(x^l, l) + (1 - \lambda)\,[u(x^h, h) - u(x^l, h) + u(x^l, l)]$$

NLP – cont.

- max $u(x^l, l) + (1 - \lambda)\,[u(x^h, h) - u(x^l, h)]$

f.o.c.

- $(1 - \lambda)\, u'(x^h, h) = 0 \rightarrow u'(x^h, h) = 0 \rightarrow$ (**no distortion at the top)**
- $u'(x^l, l) = (1 - \lambda) u'(x^l, h)$
- Problem: distortion at the bottom, the low-demand type consumes less than the efficient quantity (less than would consume at competitive price $P = 0$)
- This inefficiency is typical in mechanism design

Constrained Optimization

- As we could see in the above example, finding an optimal direct mechanism boils down to a constrained maximization problem
- The objective function represents the objectives of the principal
- There are 3 types of constraints:
  - IR: Individual Rationality (Participation) constraints – ensure that the players choose to participate in the game (do not opt out)
  - IC: Incentive Compatibility (Separation) constraints – ensure that the players indeed report their true type, and not pretend to be someone else
  - BB: Balanced Budget – ensures that the principal does not have to subsidize the mechanism, i.e. that the net sum of transfers exceeds principal's costs

An Inefficiency Theorem for double auctions

- Myerson-Satterthwaite Theorem: Suppose that the seller's and buyer's valuation have differentiable and positive densities on $[v_s^{min}, v_s^{max}]$ and $[v_b^{min}, v_b^{max}]$. Suppose also that gains from trade are possible ($v_s^{min} < v_b^{max}$), but not guaranteed ($v_s^{max} > v_b^{min}$). Then there is no fully efficient trading mechanism that satisfies IR, IC and BB.
- It can be shown, that if valuations are distributed uniformly, then the optimal (most efficient) mechanism uses the linear bid functions derived in class

A Revenue Equivalence Theorem for Auctions

- Suppose that the valuation of bidders are distributed independently symmetrically on $[v^{min}, v^{max}]$.
- All auctions that yield the same decision (assign the object to the same player-type) and give zero surplus to player-type with valuation v^{min} yield the same revenue to the principal.
- In particular, the first-price and the second-price aution are both optimal and yield the same revenue to the seller (both assign the good to the highest-valuation player)
- When valuation distributions are not symmetric, then an optimal auction does not necessarily assign the good to the highest-valuation player

An Efficiency Limit Theorem for the market game

- Wilson Theorem: The inefficiency of the market game (played in class) tends to 0 as the number of buyers and sellers tend to infinity.
- Conclusion: Larger market – more efficient market